The Turbulent Vacuum

A. P. Balachandran

Preamble

Locality and causality are intimately tied to the concept of "fields".

Faraday(1831): To resolve the "nonlocal" electromagnetic induction, he postulated the lines of force.

Maxwell(1865): He interpreted Faraday in terms of the electromagnetic field.

Since then the field concept has dominated the reconciliation of locality with causality.

Notable attempts to overcome this limitation: causal sets (Sorkin), causal dynamical triangulations (Ambjörn and Loll).

Here we study the vacuum with focus on locality and causality in relativistic quantum field theory.

The plan is to review certain remarkable features: they are not encountered for finite degrees of freedom.

In relativistic quantum field theory (QFT) the vacuum $|0\rangle$ gives the unique state $|0\rangle\langle 0|$ with zero four momentum:

$$P_\mu|0\rangle = 0.$$

It is an idealization of "nothingness".

- Vacuum is vacuous.
- When all material bodies,particles, fields are removed - we are left with the vacuum.
- It is a featureless terrain with no significant properties, colourless, odourless.

There is also another, more romantic view. It dates to QFT in its infancy.

According to this view, encouraged by perturbation theory, it is turbulent.

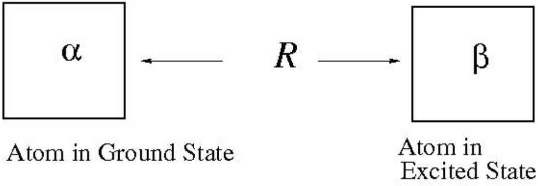
Particle and anti-particle pairs constantly appear and disappear, then kill each other off.

In the first picture, if say two detectors are inserted at spacelike distance, there should be no mutual influence.

We show that this is not the case.

This may not surprise the second camp.

An Experiment of Fermi



States $|\alpha\rangle$ and $|\beta\rangle$ are at a distance R apart. Quantum State vector at time, $t = 0$, is

$$|\Psi, t = 0\rangle = |\alpha\rangle \otimes |\beta\rangle|0\rangle_\gamma$$

Consider $t < \frac{R}{c}$. By causality, photon from the decay of $|\beta\rangle$ cannot excite $|\alpha\rangle$.

With normalised states, we infer that

$$\langle \Psi, 0| e^{iHt} (\mathbb{1} - |\chi\rangle\langle\chi|) e^{-iHt} |\Psi, 0\rangle = 0$$

where

$$|\chi\rangle = |\alpha\rangle \otimes \mathbb{1} \otimes \mathbb{1}$$

for time $t < \frac{R}{c}$
which is the probability of finding atom α in *any* excited state.

Surprise!

If this is correct, <u>either</u> $|\alpha\rangle$ will <u>never</u> get excited

or

$|\alpha\rangle$ will go <u>instantaneously</u> excited - violating causality.

Proof

$$\mathbb{Q} = \mathbb{1} - |\chi\rangle\langle\chi| = \mathbb{Q}^2$$

$$\Rightarrow \mathbb{Q}e^{-iHT}|\Psi, 0\rangle = 0, \qquad t < \frac{R}{c}.$$

If P_E projection operator to energy E ,

$$\int_{E\geq 0} d\mu(E) e^{-iEt} \mathbb{Q} P_E |\Psi, 0\rangle = 0, \qquad t < \frac{R}{c} \tag{1}$$

Thus for any state $|\Phi\rangle \in \mathcal{H}$

$$\int_{E\geq 0} \langle\Phi| e^{-iEt} \mathbb{Q} P_E |\Psi, 0\rangle d\mu(E) = 0, \qquad t < \frac{R}{c}.$$

But if $\mathrm{Im}\ t < 0$,

$$e^{-iEt} = e^{-iE\mathrm{Re}\ t} e^{EIm\ t}$$

is holomorphic.Or $(1) =$ Boundary value of a function holomorphic for $\mathrm{Im}\ t < 0$.

But it is zero for $t < \frac{R}{c}$. So it is identically zero.

If we abandon causality, (1) need not be zero. Then there is instantaneous propagation of signals!

What is wrong? We assumed

- $H \geq 0$.
- Hilbert space $\mathcal{H}$ of α, β and photon is factorizable:

$$\mathcal{H} = \mathcal{H}_\alpha \otimes \mathcal{H}_\beta \otimes \mathcal{H}_\gamma \tag{2}$$

The first one is needed for stability.
The second one is wrong for localized observables: If $\mathcal{A} =$ all observables

$$\mathcal{A} = \mathcal{A}_\alpha \otimes \mathcal{A}_\beta \otimes \mathcal{A}_\gamma$$

with $\mathcal{A}_{\alpha,\beta,\gamma}$ commuting, still (2) is false in QFT.

An Example from Quantum Mechanics

Consider two commuting rigid rotors. $\mathcal{A} =$ observables generated by

$$\mathbb{C}SO(3)_L \vee \mathbb{C}SO(3)_R \qquad \mathbb{C}SO(3)_L \cap \mathbb{C}SO(3)_R = \mathbb{C}\mathbb{1}$$

Hilbert space

$$\mathcal{H} = L^2(SO(3)) \qquad (\psi, \chi) = \int d\mu(g)\bar{\psi}(g)\chi(g)$$

$$\begin{aligned} g : \Big(U_L(g)\psi\Big)(h) &= \psi(g^{-1}h) \\ \Big(U_R(g)\psi\Big)(h) &= \psi(hg), \end{aligned}$$

with

$$\psi(h) = \sum \psi^J_{\lambda\mu} D^J_{\lambda\mu}(h)$$

Note that $\psi(h)$ does not factorize into $\mathcal{H}_L \otimes \mathcal{H}_R$.

Remark

$\mathcal{H}$ has **no** projector $\mathcal{P}(K)$ projecting to a spacetime region K if $K' =$ causal complement of $K \neq \emptyset$. I will show this.

It means :

States cannot be localised.

But first note one implication: <u>There are no localised detectors</u>.

For $|\psi_\alpha\rangle$, if a localised state, could be a localised detector.

Towards the Proof

Our first step is Reeh-Schlieder Theorem:

Let K be any spacetime region $\neq \{$ point$\}$. Then for $x_i \in K$, linear span of

$$\varphi(x_1) \cdots \varphi(x_N)|0\rangle, \qquad N = 1, 2, \cdots \tag{3}$$

approximates any vector in Hilbert space $\mathcal{H}$ arbitrarily well.

To prove:

For any $|\chi\rangle \in \mathcal{H}$,

$$\langle\chi|\varphi(x_1) \cdots \varphi(x_N)|0\rangle = 0 \tag{4}$$

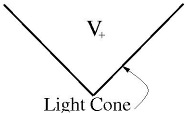
implies $|\chi\rangle = 0$.

(4) implies

$$
\begin{aligned}
& \langle \chi | e^{iP.x_1} \varphi(0) e^{-iP.x_1} e^{iP.x_2} \cdots \varphi(0) | 0 \rangle = 0 \\
\Rightarrow \quad & \int d\mu(p_i) e^{i(p_1.x_1 + \cdots - p_N.x_N)} \\
\times \quad & \langle x | \varphi(0) | p_1 \rangle \langle p_1 | \varphi(0) | p_2 \rangle \cdots \varphi(0) | p_N \rangle \langle \varphi(0) | 0 \rangle = 0 \quad (5)
\end{aligned}
$$

with p_i = total 4-momenta in vectors $|p_i\rangle$.

But $p_i \in \overline{V_+}$ = Closure of the forward light cone.



So if $\xi \in V_+$= interior of $\overline{V_+}$,

$$\xi . p_i > 0$$

So if

$$\begin{aligned} x_1 &= \mathrm{Re}\ x_1 + i\xi_1 \\ x_1 - x_2 &= \mathrm{Re}\ x_1 - x_2 - i\xi_2 \\ &\vdots \end{aligned}$$

and $p_i.\xi_i > 0$, one gets (5) being holomorphic.

If its boundary value $= 0$, it is identically 0.

$$\Rightarrow \langle \chi | \varphi(x_1) \cdots \varphi(x_N) | 0 \rangle = 0 \qquad \forall x_i \text{ and } N$$

But such $\varphi(x_1) \cdots \varphi(x_N)|0\rangle$ generate all $\mathcal{H} \Rightarrow |\chi\rangle = 0$

QED

Instead of vacuum, we can use any vector $|\psi\rangle \in \mathcal{H}$ of finite energy in above theorem.

No Projectors!

Now we proceed to our result:

If K is a spacetime region, K' its causal complement $\neq$\{ point\},

the observables localised on $K, K' : \mathcal{A}_K, \mathcal{A}_{K'}$ commute$\Rightarrow$

$$\mathcal{A}_{K \cup K'} = \mathcal{A}_K \vee \mathcal{A}_{K'} = \mathcal{A}$$

which is a "bipartite system".

This tempts us to guess that Hilbert spaces also behave in a similar way:

$$\mathcal{H}_{K \cup K'} \stackrel{?}{=} \mathcal{H}_K \otimes \mathcal{H}_{K'}$$

The result we find is: NO!

How to defeat Fermi?

The Reeh-Schlieder theorem tells us that $\varphi(x_1)\cdots\varphi(x_N)|\psi\rangle$, with $x_i \in K$ or K' both give the same full Hilbert space! Thus there are no projectors $\mathcal{P}(K), \mathcal{P}_{K'}$ to K and K'.

Reeh-Schlieder subdues Fermi:

$$\boxed{\mathcal{H} \neq \mathcal{H}_\alpha \otimes \mathcal{H}_\beta \otimes \mathcal{H}_\gamma}$$

On Partial Traces over states, Functional Integrals

In QFT, as localised states do not exist, partial traces to compute entropy etc. need reexamination.
Functional integrals too, when used to calculate say transition amplitudes between states need to be critically examined.

Coleman Theorem

Symmetry generators, like ,

$$\text{Charge} \qquad Q = \int d^3x \, j_0(x)$$

$$\text{Axial Charge} \qquad Q^5 = \int d^3x \, j_0^5(x)$$

are integrals of local densities $j_0(x), j_0^5(x)$.

Note: the quantities like $[H, j_0(x)]$ are local fields - where H is the Hamiltonian.

Theorem(Coleman)

If say Q^5 annihilates vacuum, it commutes with H: Invariance of the vacuum is the invariance of the world.

Proof:

If

$$\int j_0 \; d^3x|0\rangle = 0, \; \rightarrow \langle\{p_i\}| \int d^3x \; j_0(x)|0\rangle = 0$$

With $P = \sum_i p_i, \; j_0(x) = e^{iP.x} j_0(0) e^{-iP.x}$, we get

$$\delta^3(P)\langle\{p_i\}|j_0(0)|0\rangle = 0, \; \Rightarrow \langle\{p_i\}|j_0(0)|0\rangle|_{P=0} = 0$$

Proof(Contd.)

$$\langle\{p_i\}|\partial^\lambda j_\lambda(0)|0\rangle|_{P=0} = 0$$

since $\partial^i j_i(0) = [P_i, J_i(0)]$.

Hence

$$\langle\{p_i\}|\partial^\lambda j_\lambda(x)|0\rangle|_{P=0} = 0$$

by

$$\partial^\lambda j_\lambda(x) = e^{iP.x}\partial^\lambda j_\lambda(0)e^{-iP.x}$$

But any state $|\{p_i\}\rangle$ can be transformed to $|\{p_i\}\rangle_{P=0}$ since $P \in V_+$.

So,

$$\langle \{p_i\} | \partial^\lambda j_\lambda(x) | 0 \rangle |_{P=0} = 0$$

$\Rightarrow$

$$\partial^\lambda j_\lambda(x) | 0 \rangle$$

But $\partial^\lambda j_\lambda$ is a local field $\Rightarrow$ By Reeh-Schlieder,

$$\partial^\lambda j_\lambda(x) = 0 \Rightarrow \partial^0 \int j_0(x)\ d^3x = i[P_0, \int j_0(x)\ d^3x] = 0$$

So symmetry of the vacuum is the symmetry of the world.

We can combine this theorem with the Fabri-Picasso theorem

The integral

$$Q(t) = \int d^3x \ j(x)$$

of a local field *either diverges on the vacuum* or annihilates it (so is an exact symmetry).

The proof assumes that the vacuum is isolated in the spectrum of P_μ. Proof

$$\langle 0|Q(t)Q(t)|0\rangle = \int d^3x \ \langle 0|J(x)Q(t)|0\rangle = \infty$$

unless

$$Q(t)|0\rangle = 0!$$

QCD seems to avoid this theorem since $\{0\}$ is not isolated in spectrum P_μ.

In spontaneous breakdown also $\{0\}$ is not isolated due to the Goldstone modes.

Conclusion: In non-relativistic quantum mechanics we have:

Born Localisation.

Born Localisation

If K, K' spatial regions at same time with projectors $P_K, P_{K'}$, and

$$K \cap K' = \emptyset \Rightarrow$$

$|\psi_K\rangle = \mathcal{P}(K)|\psi\rangle, |\chi_{K'}\rangle = \mathcal{P}(K')|\chi\rangle$, are localised orthogonally in K, K' :

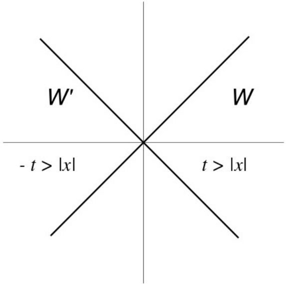
$$\langle\psi_K|\chi_{K'}\rangle = 0 \qquad \text{Orthogonal Localisation}$$

Born localisation fails in QFT with causality.
We only have causal or symplectic localisation of observables:

$$\alpha_K \in \mathcal{A}_K, \beta_{K'} \in \mathcal{A}_{K'} \Rightarrow [\alpha_K, \beta_{K'}] = 0$$

The Rindler Wedge

Now let us look at the behaviour of QED on the Rindler wedge.



W, W' shown above are causal complements $\Rightarrow$

- $[\alpha_W, \beta_{W'}] = 0$ where $\alpha_W \in \mathcal{A}_W =$ observables in W.
- As we saw ,

$$\mathcal{A}_{W \cup W'} = \mathcal{A}_W \vee \mathcal{A}_{W'}, \qquad \mathcal{A}_W \cap \mathcal{A}_{W'} = \mathbb{C}\mathbb{1}$$

Example: For real scalar fields φ, we have observables

$$\{\varphi(f_W) \equiv \int d^4x \, \varphi(x) f_W(x) : f_W = f_W^* \in \mathcal{C}_0^\infty(W)\}$$

$$\Rightarrow \qquad [\varphi(g_{W'}), \varphi(f_W)] = 0$$

This is "symplectic localisation".

Attention to gauge Theories

Covariant Gauss Law • Work with Asorey, Lizzi , Marmo

We consider free fields:

In fixed time quantisation , Gauss law is

$$\partial_i E_i : \partial_i E_i |\cdot\rangle = 0$$

which is a first class constraint.

But quantum fields are distributions , so we must write

$$\int \partial_i \Lambda E_i |0\rangle = 0. \qquad \Lambda \in \mathcal{C}_0^\infty(\mathbb{R}^3).$$

which is still unsatisfactory.

Recall that quantum fields are *spacetime* distributions - they cannot be restricted to a given time. Do we have a $\boxed{\text{spacetime}}$ formulation of QED?

Yes!

The spacetime observables are

$$A(\varphi) = \int d^4x \; A_\mu(x)\varphi^\mu(x), \qquad \partial_\mu \varphi^\mu = 0, \qquad \varphi \in \mathbb{C}_0^\infty(\mathbb{R}^4).$$

$A(\varphi)$ is gauge invariant since

$$\int d^4x \; (\partial_\mu \Lambda)(x)\varphi^\mu(x) = 0, \quad \forall \Lambda \in \mathcal{C}_0^\infty(\mathbb{R}^4).$$

The observable algebra is generated by $A(\varphi)$ with

$$[A(\varphi), A(\chi)] = \int d^4x\ d^4y\ \varphi^\mu(x) D(x-y) \chi_\mu(y)$$

with $D(x-y)$ being the causal commutator

$$\int d\mu(k)[e^{-ik.(x-y)} - c.c], \qquad d\mu(k) \equiv \frac{d^3k}{(2\pi)^3(2k_0)}, \qquad k_0 = |\mathbf{k}|$$

Eq. of motion is a constraint

$$G(\eta) = \int d^4x \; \partial^\lambda F_{\lambda\mu}(\eta) A^\mu(x), \qquad \eta_\mu \in \mathcal{C}_0^\infty(\mathbb{R}^4),$$

$$G(\eta)|.\rangle \;=\; 0$$

But $\partial^\mu \eta_\mu$ need not be zero.
For, classically ,

$$G(\eta) = \int d^4x \; \eta^\mu(x) \partial_\lambda F_{\lambda\mu}(A)(x)$$

is zero by E.O.M.

Physical Meaning of $G(\eta)$

$G(\eta)$ generate spacetime gauge transformations:

$$\begin{aligned} [G(\eta), A_\rho(y)] &= -\frac{\partial}{\partial y^\rho} \int d^4x \, (\partial.\eta)(x) D(x-y) \\ &\Rightarrow [G(\eta), A(\varphi)] = 0 \end{aligned}$$

as $\partial^\rho \varphi_\rho = 0, \varphi \in \mathcal{C}_0^\infty(\mathbb{R}^4)$.

Superselected operators are obtained from

$$Q(\zeta) = \int d^4x \, \left[\partial^\lambda F_{\lambda\mu}(\zeta)\right](x) A^\mu(x)$$

where $\zeta \in \mathcal{C}^\infty(\mathbb{R}^4)$ but need not be $\mathcal{C}_0^\infty(\mathbb{R}^4)$..

But on quantum states $|.\rangle$, $G(\eta)|.\rangle = 0$.

That is also the case for $A(\varphi)|F\rangle$, with $|F\rangle$ being any Fock state and φ, η are localised (supported) in W .

So, with no infrared effects, $\mathcal{A}_W|F\rangle$ supports E.O.M.

Infra dress: Non-Fock states

Now consider $G(\eta)$:

$$[G(\eta), A_\rho(x)] = -\partial_\mu \int_W d^4y \ (\partial.\eta)(y)D(y-x)$$

for $\mathrm{Supp}(\eta) \in W$.
Hence consider as warm-up

$$\begin{aligned} & G(\eta)e^{-i\int_{-\infty}^{0} dx^\mu \ A_\mu(x(t))}|P.Q, \cdots\rangle|0\rangle_\gamma \\ = \quad & e^{-i\int_{-\infty}^{0} dx^\mu A_\mu(x(t))} \\ \times \quad & i\partial_t \int_W d^4y \ \partial.\eta(y)D(y-x(t))]|P,Q,..\rangle|0\rangle_\gamma \end{aligned}$$

with $x(t) = \frac{P}{m}t$

But

$$D(y - x(t)) \neq 0$$

for $x(t) \in \mathbb{R}^4 \setminus W'$.

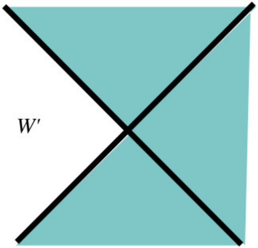


Figure: $\mathbb{R}^4 \setminus W'$ is shaded above

$\Rightarrow$ Photon EOM cannot be localised in W.

There is information loss.

But now for a charged particle , there is an infrared cloud.

The dressed particle is in a non-Fock space.

If particle has momentum p and charge e, the dressed charged state is

$$e^{-i\int_{-\infty}^{0} dt\,\frac{p^\mu}{M}\;A_\mu(\frac{p}{M}t)}|p,e\rangle|0\rangle_\gamma$$

where states $|p,e\rangle, |0\rangle_\gamma$ are in Fock space.

For N such particles of total momentum

$$P = \sum_i p^{(i)}$$

and charge

$$Q = \sum_i e^{(i)}$$

the above changes to

$$e^{-i\int_{-\infty}^{0} dt \frac{P^\mu}{M} \ A_\mu(\frac{P}{M}t)} |P, Q\rangle |0\rangle_\gamma = e^{-i\int_{-\infty}^{0} dx^\mu A_\mu(x(t))} |P, Q\rangle |0\rangle_\gamma$$

where the last factor is a timelike Wilson line integral.

Note

We start with a representation of all observables $\mathcal{A}$.

Then we localise $\mathcal{A}$ to $\mathcal{A}_W$.

We cannot localise states.

Remark

This can be extended to Chern-Simons in $2 + 1$ dimensions.

Gravity's Rainbow

Can we extend these considerations to quantum gravity?

Gravity is universal.

This extension will show that there is always information leakage from Rindler wedge.

That would suggest a similar result for black holes.

A short introduction to Fierz-Pauli

We will work in linearized gravity: the equations are those of Fierz and Pauli - so we introduce them.
"Classical" functions will have no hats.
Let us set

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

where $h_{\mu\nu}$ is small. The "classical" fields will serve as test functions.
Quantum operators will have hats.
The Fierz-Pauli Lagrangian density $\mathcal{L}$ for spin-2 is of the formulation

$$\int \delta\mathcal{L}\ d^4x = \int \mathcal{H}^{\mu\nu}(h)\delta h_{\mu\nu}\ d^4x$$

which gives the identity

$$\partial_\mu \mathcal{H}^{\mu\nu}(h) = 0.$$

Given

$$\delta h_{\mu\nu} = \partial_{(\mu}\chi_{\nu)}$$

one gets

$$\int \delta\mathcal{L}\ d^4x = 0.$$

which is the gauge invariance for this system.

The classical equation of motion is,

$$\mathcal{H}^{\mu\nu} = 0$$

The gauge condition is

$$\partial_\lambda(h^{\lambda\mu} - \frac{1}{2}\eta^{\lambda\mu}h^\sigma{}_\sigma) \equiv \partial_\lambda \bar{h}^{\lambda\mu} = 0 \qquad (6)$$

Quantum Observables

Local observables

$$\hat{h}(\bar{h}) = \int d^4x \; \bar{h}^{\mu\nu}\hat{h}_{\mu\nu}(x)$$

$\bar{h}^{\mu\nu} \in C_0^\infty(\mathbb{R}^4)$ is gauge invariant by (6).

Quantum Algebra (can be derived):

$$[\hat{h}^{\mu\nu}(x), \hat{h}^{\alpha\beta}(y)] = \left[\frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + (\alpha \leftrightarrow \beta) - \frac{1}{4}\eta^{\mu\nu}\eta^{\alpha\beta}\right] D(x-y)$$

with

$$\Box D(x-y) = 0$$

$D(x)$ being the Pauli-Jordan function , vanishing for spacelike arguments.

The next step is equation of motion as constraints. They are

$$\hat{G}(\mathcal{H}) = \int d^4x \; \mathcal{H}_{\mu\nu}(h)\hat{h}^{\mu\nu}$$

where $\mathcal{H}_{\mu\nu}(h)$ is equation of motion.

Since $\partial^\mu \mathcal{H}_{\mu\nu} = 0$, $\mathcal{H}_{\mu\nu} =$ a test function $\bar{h}_{\mu\nu}$.

The constraints are

$$\hat{G}(\mathcal{H})|.\rangle = 0$$

For by partial integrations

$$\hat{G}(\mathcal{H}) = \int d^4x \; h_{\mu\nu}\mathcal{H}^{\mu\nu}(\hat{h})$$

so that $G(\mathcal{H}) = 0$ gives the classical equations.

Now:

- The constraints generate gauge transformations.
- Hence they commute with observables.

  and
- They are first class.

Proof:

$$[\hat{G}(\mathcal{H}, \hat{h}_{\alpha\beta}(y)] = \int d^4x \, (\mathcal{H}_{\alpha\beta}(x) - \frac{1}{4}\eta_{\alpha\beta}\mathcal{H}(x))D(x-y)$$

Using $\Box D(x-y) = 0$ we get

$$[\hat{G}(\mathcal{H}, \hat{h}_{\alpha\beta}(y)] = \partial_\alpha \chi_\beta(y) + \partial_\beta \chi_\alpha(y)$$

with

$$\chi_\alpha(y) = \int d^4x \, (\frac{1}{2}\partial_\alpha h - \partial^\rho h_{\alpha\rho}(x))D(x-y)$$

as required.

We have now recovered the analogue of QED equation *except* the infragraviton twist .

For this we need a coupling

$$\int d^4x \; J^{\mu\nu}(x)\bar{h}_{\mu\nu}(x)$$

where

1. $J^{\mu\nu}$ is a conserved classical source $\Rightarrow$ interaction is gauge invariant.
2. It is for a point particle, valid for low graviton emission.

For momentum p, one has

$$J^{\mu\nu}(x) = \int d\tau \; p^{\mu}p^{\nu}\delta^4(x - \frac{p}{m}\tau) \Rightarrow \partial_{\mu}J^{\mu\nu} = 0$$

For infraradiation, backreaction can be neglected

$\Rightarrow p^{\mu}$ constant.

For a particle of mass m, $p^2 = m^2$, the *in* state as before is,

$$|p_i, 0\rangle_{in} = \exp\left[i \int_{x_0<0} dx_0 \; d^3x \; J^{\mu\nu}(x)\hat{h}_{\mu\nu}(x)\right] |p\rangle|0\rangle_h$$

where

- $|p\rangle$ = Radiating particle of Fock state
- $|0\rangle_h$ = Graviton Fock ground state

The commutators of $\hat{\mathcal{H}}$ involve D-function. Hence as before if $\hat{G}(\mathcal{H})$ is appropriate for W,

$$\hat{G}(\mathcal{H}) = \int_W d^4x \; \mathcal{H}^{\mu\nu}(x)\hat{h}_{\mu\nu}(x),$$

still we find

$$ {}_{in}\langle p;0|\hat{G}(\mathcal{H})|p;0\rangle_{in} \tag{7} $$

does not vanish for $x(t) = \frac{p}{m}t \notin W$:

(7) involves $D(x - \frac{p}{m}t), x \in W$.

We can take for W,

$$ \tilde{W} \subset W $$

and $x(t)$ entirely outside $\tilde{W}$. Still (7) vanishes only for $x(t) \in W'$.

Work in Progress : Generalisation to Any Massless Integral Spin Particle

Consider a massless helicity $s \in \mathbb{Z}$ particle. Equations of motion are those of Fronsdal. [1]

Superselection Operators

For QED, the current J_ν leads to a scalar charge Q as a superselection operator.

So, we guess and can prove that for Fierz-Pauli-Fronsdal: it is momentum P_μ.

For spin 3, it will be a symmetric traceless tensor $Q_{\mu\nu}$.

Thus we expect a hierarchy $Q, Q_\mu = P_\mu, Q_{\mu\nu}, \cdots$.

Open Questions:

- What is the interpretation of $Q_{\mu\nu}$?
- Their algebras?

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[1] Elegantly discussed by Asorey et.al.